

# Quark confinement due to creation of micro AdS black holes in quarkonium model

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November 17, 2016

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## Abstract

We use the solution of the Dirac equation for quarkonium atom in the 4D Anti de sitter (AdS<sub>4</sub>) space to investigate the effect of the large negative cosmological constant on the phenomenon of quark confinement. We do the required calculations in the AdS<sub>4</sub> space to indicate that large cosmological constant can describe the quark confinement. In fact using the coulomb potential in Dirac equation while we employ the AdS metric will additionally lead us to a linear potential in the quark-antiquark interaction which can be considered to explain the quark confinement. This confining term is arising essentially from the geometrical features of the space. On the other hand the origin of the large cosmological constant can be justified by assuming the appearance of micro black holes in the recent hadronic collision process which is now current, for instance, at the LHC project.

**Keywords:** Dirac equation, Quarkonium, Anti de sitter space, Quark confinement, Micro black hole.

## 1 Introduction

It has been known for long time that due to quark confinement and the short range of the strong force, the production of quark anti-quark pair in the  $e^+ e^-$  collision will result to hadronic jets instead of individual colored quarks. But the mechanism of this confinement is still ambiguous. There were many attempts to explain the colour confinement such as bag models [1, 2, 3, 4], confinement potential models [5, 6], quasi particle models [7], strongly interacting quark-gluon-plasma (sQGP) [8, 9, 10] and etc. In spite the many attempts which have been done but people still do not reach to an unique and acceptable framework on this issue.

In this work we investigate the effect of the four dimensional (4D) Ads metric on the quarkonium atom that its coloured quark-antiquark interaction is described only by Coulomb-like potential  $V(r) = \frac{a}{r}$ . As we will see the existence of an Ads metric with a negative cosmological constant impose the confining term  $\frac{\Lambda}{6}ar$  in the Dirac equation which is added to the coulomb potential term. In comparison with other confining models, the new effective potential  $V_{eff}(r) = \frac{a}{r} - \frac{\Lambda}{6}ar$  acts like the Cornell potential [11]. Cornell potential models that are used to describe the mass spectrum of heavy mesons have been established on a non-relativistic potential  $V_{Cornell}(r) = \frac{a}{r} - br$ , where its Coulomb part is a non-relativistic limit of the one-gluon exchange interaction dominant at short distance interactions, while the linear part comes from the Wilson loop and ensures the confinement of quarks. As an interesting result, using the Ads metric we achieve the quark confinement in a hydrogen-like atom which is actually resulted from the geometrical properties of space.

By comparing the unknown constants in effective potential with the Cornell potential whose parameters are determined via fitting the experimental data, we obtain the numerical value of cosmological constant as

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$\frac{\Lambda}{6} \approx 10^{30} m^{-2}$ . Such a large cosmological constant can be produced by a black holes in its nearby space. By new hypotheses, micro black holes could be performed at an energy scale of the  $TeV$  order [12] which are available now in recent colliding machines at CERN. When one try to de-confine the quarks in the quarkonium atom then the creation of Ads micro black holes is expected [13, 14].

It is known that for a Schwarzschild black hole in an AdS space, the black hole is thermodynamically unstable when the horizon radius is small [15]. So, in contrast with normal black holes that have a long life time, micro black holes have a very short life time of  $10^{-26}s$  order. This means that once they will be created they will evaporate instantaneously. However, as we will discuss the creation of AdS micro black holes while trying to de-confine the quarks can explain the quark confinement.

On the other hand, there is an inspiring and different idea which was proposed by E.Witten that tried to explain the connection between geometry of space and the quark confinement problem via the Ads space [16]. This can be considered as a confirmation to what we do to employ the Ads metric to extract the linear potential which is justifying the quark confinement. At the present time, the developed AdS/QCD approach to hadron physics provides remarkable descriptions in many phenomenological features of QCD.

The outline of the paper is as follows. In section 2, we introduce the Dirac Equation on Anti de sitter space. The Solution of Dirac equation for quarkonium atoms, using the Ads metric is described in Section 3 which lead us to an additional linear potential. We give finally our conclusion in section 4.

## 2 The Dirac equation on AdS metric

The  $AdS_4$  metric in a global coordinates is defined by

$$g_{\mu\nu} dx^\mu dx^\nu = F(r) dt^2 - \frac{1}{F(r)} dr^2 - r^2 (d\theta^2 + \sin^2 \theta d\phi^2), \quad (1)$$

where  $F(r) = 1 + \frac{\Lambda}{3} r^2$  [15].

We consider the covariant form of Dirac equation on the metric  $g_{\mu\nu}$  that has the form

$$i\gamma_{(g)}^\mu \nabla_\mu \psi - m\psi = 0. \quad (2)$$

In this case  $\gamma_{(g)}^\mu$  are the contravariant Dirac matrices and  $\nabla_\mu$  is the covariant derivative which are satisfying the following relations [17, 18]:

$$\gamma_{(g)}^\mu = \gamma_\alpha e^{\alpha\mu} \quad \alpha = 1, 2, 3, \quad (3)$$

$$\nabla_\mu = \partial_\mu + \frac{1}{8} [\gamma_\alpha, \gamma_\beta] \omega_\mu^{\alpha\beta},$$

where  $\gamma_\alpha$  is the ordinary Dirac matrices such that:

$$\gamma^0 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}, \quad \gamma^\alpha = \begin{pmatrix} 0 & \sigma^\alpha \\ -\sigma^\alpha & 0 \end{pmatrix}, \quad (4)$$

in which  $\sigma^\alpha$  and  $I$  are  $2 \times 2$  Pauli and identity matrices respectively. The coefficient of spin connection  $\omega_\mu^{\alpha\beta}$  as a function of vierbein fields  $e_\alpha^\mu$  is given by [17]

$$\omega_\mu^{\alpha\beta} = \frac{1}{2} e^{\alpha\nu} (e_{\nu,\mu}^\beta - e_{\mu,\nu}^\beta) - \frac{1}{2} e^{\beta\nu} (e_{\nu,\mu}^\alpha - e_{\mu,\nu}^\alpha) + \frac{1}{2} e^{\alpha\nu} e^{\beta\sigma} (e_{\nu,\sigma}^\rho - e_{\sigma,\nu}^\rho) e_{\rho\mu}. \quad (5)$$

However, if we choose the local orthonormal Lorentz frame, the Dirac equation with electric potential  $V(r)$  has the following form in spherical coordinate system [see e.g. [18]].

$$\left[ iF^{-\frac{1}{2}} \gamma^0 \left( \frac{\partial}{\partial t} + iV(r) \right) + iF^{\frac{1}{2}} \gamma^1 \left( \frac{\partial}{\partial r} + \frac{1}{r} + \frac{F'}{4F} \right) + \frac{i}{r} \gamma^2 \left( \frac{\partial}{\partial \theta} + \frac{1}{2 \tan \theta} \right) + \frac{i}{r \sin \theta} \gamma^3 \frac{\partial}{\partial \phi} - M \right] \psi(r, \theta, \phi, t) = 0. \quad (6)$$

Now, if we put  $\psi(r, \theta, \phi, t) = \left(r (\sin \theta)^{\frac{1}{2}} F^{\frac{1}{4}}\right)^{-1} \varphi(r, \theta, \phi, t)$ , the Eq. (6) can be rewritten in a simple form as it follows

$$\left[ iF^{-\frac{1}{2}} \gamma^0 \left( \frac{\partial}{\partial t} + iV(r) \right) + iF^{\frac{1}{2}} \gamma^1 \left( \frac{\partial}{\partial r} \right) + \frac{i}{r} \gamma^2 \left( \frac{\partial}{\partial \theta} \right) + \frac{i}{r \sin \theta} \gamma^3 \frac{\partial}{\partial \phi} - M \right] \varphi(r, \theta, \phi, t) = 0. \quad (7)$$

It is convenient to make a separation of variables in Eq. (7), choosing

$$\varphi(r, \theta, \phi, t) = \frac{1}{r} \begin{pmatrix} f(r) Y_{lm}(\theta, \phi) \\ ig(r) Y_{l'm}(\theta, \phi) \end{pmatrix} e^{-iEt}. \quad (8)$$

By substituting Eq. (8) into Eq. (7) and considering this fact that the angular part of the Dirac equation in AdS space, given by Eq. (1), is the same as Dirac equation in usual space, we will arrive at

$$\begin{aligned} \left[ M - F^{-\frac{1}{2}} (E - V(r)) \right] f(r) &= F^{\frac{1}{2}} g'(r) - \frac{\kappa}{r} g(r), \\ \left[ M + F^{-\frac{1}{2}} (E - V(r)) \right] g(r) &= F^{\frac{1}{2}} f'(r) + \frac{\kappa}{r} f(r), \end{aligned} \quad (9)$$

where  $\kappa = \pm (j + 1/2)$  [12].

### 3 The Solution of Dirac equation for quarkonium atom and quark confinement

Considering the short distance behavior of quark-antiquark interaction with order of  $r \sim 10^{-16}m$ , we then are able to solve the Eq. (9). For this purpose, we use the Taylor expansion of  $F^{\frac{1}{2}}$  and  $F^{-\frac{1}{2}}$  terms in Eq. (9) as it follows

$$\begin{aligned} F^{\frac{1}{2}} &= \left(1 + \frac{\Lambda}{3} r^2\right)^{\frac{1}{2}} \approx 1 + \frac{\Lambda}{6} r^2 - \frac{\Lambda^2}{72} r^4 + O(r^5), \\ F^{-\frac{1}{2}} &= \left(1 + \frac{\Lambda}{3} r^2\right)^{-\frac{1}{2}} \approx 1 - \frac{\Lambda}{6} r^2 + \frac{\Lambda^2}{24} r^4 + O(r^5). \end{aligned} \quad (10)$$

Substituting these expansions into Eq. (9) and ignoring second and higher order terms with respect to  $r$  while using Coulombian-like potential  $V(r) = \frac{a}{r}$  for quark-antiquark interaction, we will get

$$\begin{aligned} \left[ M - \left( E - \left( \frac{a}{r} - \frac{\Lambda}{6} ar \right) \right) \right] f(r) &= g'(r) - \frac{\kappa}{r} g(r), \\ \left[ M + \left( E - \left( \frac{a}{r} - \frac{\Lambda}{6} ar \right) \right) \right] g(r) &= f'(r) + \frac{\kappa}{r} f(r). \end{aligned} \quad (11)$$

The appearance of the  $\frac{\Lambda}{6} ar$  term that has been added to the Coulomb potential in Eq. (11) is equal to confinement term in Cornell potential which is used to describe the quark confinement that involves the Coulomb potential " $\frac{a}{r}$ " plus a linear term " $br$ " i.e.

$$V_{\text{Cornell}}(r) = \frac{a}{r} - br, \quad (12)$$

where  $a$  is a parameter representing the Coulomb strength, and  $b$  measures the strength of the linear confining term. Now, by comparing the Cornell potential with the confinement potential that is induced by the AdS metric on Dirac Eq.(11), one can reach to an expression for cosmological constant  $\Lambda$  in terms of potential constants  $a$  and  $b$  in Eq. (12) as following

$$\frac{\Lambda}{6} = \frac{b}{a}. \quad (13)$$

Finding the proper value of  $a$  and  $b$  parameters in Cornell potential that produce the spectrum of quarkonium atom, had been the main subject of many works [19, 20, 21, 22]. In spite of small differences between the

value of parameters  $a$  and  $b$  in different works, in all of them the ratio  $\frac{b}{a}$  is about  $4 \times 10^{30} m^{-2}$ . So, we can obtain the approximate value of  $\frac{\Lambda}{6}$  term, i.e.

$$\left| \frac{\Lambda}{6} \right| \approx 4 \times 10^{30} m^{-2}. \quad (14)$$

In addition, the radial function  $f(r)$  in Eq.(11) for Cornell potential can be defined as [12]

$$f(r) = Ai \left( \left( \frac{\Lambda}{6} a \right)^{1/3} + a_n \right) P_{n'}(r) e^{-\eta r} r^l, \quad (15)$$

where  $Ai(r)$  is the Airy function and  $a_n$  are its roots. In Eq.(15)  $P_{n'}(r)$  is representing the Legendre function and  $\eta$  is a function of quarkonium energy states which is discussed in [12].

It is obvious that such a large cosmological constant can be just produced by a black holes in its nearby space. Furthermore, by new hypotheses that micro black holes could be performed at the energy scales of  $TeV$  order which are appropriate to destroy the quarkonium systems, one can expect the creation of AdS micro black holes to describe the quark confinement through the discussed method.

To check the possibility of the creation of micro black holes inside the quarkonium atom we should calculate the horizon radius of black hole that that is imposed by the cosmological constant, determined in above. For the  $D$ -dimensional AdS black holes the maximum event horizon is defined by [23, 24]

$$r_{max}^2 = - \left( \frac{D-3}{D-1} \right) \frac{3}{\Lambda}. \quad (16)$$

For  $D = 4$  and  $\frac{\Lambda}{6} \approx 4 \times 10^{30} m^{-2}$  the maximum value of event horizon radius is about  $r_{max} \approx 2 \times 10^{-16} m$ . By comparing this radius with the radius of the mesons which is determined through the de-confining process, we can conclude that the creation of micro black holes inside mesons is completely possible.

## 4 Conclusion

The main purpose of our work was the investigation of the effect of negative cosmological constant in the Anti de sitter space on the binding states of quarkonium atoms. We proved that the 4D AdS metric with large cosmological constant can explain the quark confinement by imposing a confining term in the quark-antiquark interaction potential.

Furthermore we discussed that in the case of large cosmological constant, the appearance of micro black holes could be considered as an alternative explanation for quark confinement phenomenon in the hadron collision. To get the exact solution of the Dirac equation, using the AdS metric for quarkonium atom is a valuable subject which we hope to report on them as our new research task in future.

We should mention that in spite to ignore the higher order terms in Eq.(9), this assumption do not disturb our final result. In fact the appearance of extra power terms in the right side of Eq.(9) can be absorbed into wave function while these extra terms in the left side of this equation play the role of harmonic terms in effective potential. This means that we will have a stronger confining potential with respect to the Cornell potential which are now containing two linear and quadratic order of  $r$  terms [23, 24]. Investigating this confining potential and the results which are arising it, can be considered as our further research activity.

## References

- [1] A. Chodos, R. L. Jaffe, K. Johnson, C. B. Thorn and V. F. Weisskopf, Phys. Rev. D9 (1974) 3471.
- [2] A. Chodos, R. L. Jaffe, K. Johnson, C. B. Thorn, Phys. Rev. D10 (1974) 2599.

- [3] T. DeGrand, R. L Jaffe, K. Johnson and J. Kiskis, Phys. Rev. D12 (1975) 2060.
- [4] K. Johnson and C. B. Thorn, Phys. Rev. D13 (1976) 1934.
- [5] E. Eichten, K. Gottfried, T. Kinoshita, K. D. Lane and T. M. Yan, Phys. Rev. D17 (1978) 3090.
- [6] E. Eichten, K. Gottfried, T. Kinoshita, K. D. Lane, and T. M. Yan, Phys. Rev. D21 (1980) 203.
- [7] A. Peshier, B. Kmpfer, O. P. Pavlenko and G. Soff, Phys. Lett. B337 (1994) 235.
- [8] M. Gyulassy and L. McLerran, Nucl.Phys. A750 (2005) 30.
- [9] E. V. Shuryak, Nucl.Phys. A750 (2005) 64.
- [10] B. Muller and J. L. Nagle, Ann.Rev.Nucl.Part.Sci 56 (2006) 93.
- [11] L. A. Trevisan, C. Mirez, and F. M. Andrade, Few Body Syst. 55 (2014) 1055.
- [12] M. Cavaglia, Int. J. Mod. Phys. A 18 (2003) 1843.
- [13] R. J. Adler and D. I. Santiago, Mod. Phys. Lett. A14 (1999) 1371.
- [14] G. L. Alberghi, R. Casadio, O. Micu, A. Orlandi, JHEP 1109 (2011) 023.
- [15] S. W. Hawking and D. N. Page, Commun. Math. Phys 87 (1983) 577.
- [16] E. Witten, current science 81 (2001) 1576.
- [17] J. P. Nicolas, In Annales de l'IHP Physique thorique 62 (1995) 145.
- [18] A. Bachelot, Commun. Math. Phys. 283 (2008) 127.
- [19] R. L. Hall, N. Saad, Open Phys. 13 (2015) 83.
- [20] Sameer M. Ikhdaire, Adv. High Energy Phys (2013) 491648.
- [21] A. Vega, and J. Flores, arXiv:1410.2417.
- [22] F. Karsch, M.T. Mehr, H. Satz, Zeitschrift fr Physik C Particles and Fields, 37 (1988) 617.
- [23] A.F. Al-Jamel, H. Widyan, Applied Physics Research 4 (2012) 94.
- [24] G. Bhanot, S. Rudaz, Phys. Lett. B 78 (1978) 119.